# The Matrix as Metaphysics

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# **Brains in Vats**

*The Matrix* presents a version of an old philosophical fable: the brain in a vat. A disembodied brain is floating in a vat, inside a scientist's laboratory. The scientist has arranged that the brain will be stimulated with the same sort of inputs that a normal embodied brain receives. To do this, the brain is connected to a giant computer simulation of a world. The simulation determines which inputs the brain receives. When the brain produces outputs, these are fed

back into the simulation. The internal state of the brain is just like that of a normal brain, despite the fact that it lacks a body. From the brain's point of view, things seem very much as they seem to you and me.

The brain is massively deluded, it seems. It has all sorts of false beliefs about the world. It believes that it has a body, but it has no body. It believes that it is walking outside in the sunlight, but in fact it is inside a dark lab. It believes it is one place, when in fact it may be somewhere quite different. Perhaps it thinks it is in Tucson, when it is actually in Australia, or even in outer space.

Neo's situation at the beginning of *The Matrix* is something like this. He thinks that he lives in a city; he thinks that he has hair; he thinks it is 1999; and he thinks that it is sunny outside. In reality, he is floating in a pod in space; he has no hair; the year is around 2199; and

This paper was written for the "philosophy section" of the official Matrix website. It also appears in C. Grau, ed, *Philosophers Explore the Matrix*, Oxford University Press, 2005. As such, the bulk of the paper is written to be accessible for an audience without a background in philosophy. At the same time, this paper is intended as a serious work of philosophy, with relevance for central issues in epistemology, metaphysics, and the philosophy of mind and language. A section of "philosophical notes" at the end of the article draws out some of these connections explicitly.

the world has been darkened by war. There are a few small differences from the vat scenario above: Neo's brain is located in a body, and the computer simulation is controlled by machines rather than by a scientist. But the essential details are much the same. In effect, Neo is a brain in a vat.

Let's say that a *matrix* (lower-case m) is an artificially designed computer simulation of a world. So the Matrix in the movie is one example of a matrix. And let's say that someone is *envatted*, or that they are *in a matrix*, if they have a cognitive system which receives its inputs from and sends its outputs to a matrix. Then the brain at the beginning is envatted and so is Neo.

We can imagine that a matrix simulates the entire physics of a world, keeping track of every last particle throughout space and time. (Later, we will look at ways in which this setup might be varied.) An envatted being will be associated with a particular simulated body. A connection is arranged so that whenever this body receives sensory inputs inside the simulation, the envatted cognitive system will receive sensory inputs of the same sort. When the envatted cognitive system produces motor outputs, corresponding outputs will be fed to the motor organs of the simulated body.

When the possibility of a matrix is raised, a question immediately follows: how do I know that I am not in a matrix? After all, there could be a brain in a vat structured exactly like my brain, hooked up to a matrix, with experiences indistinguishable from those I am having now. From the inside, there is no way to tell for sure that I am not in the situation of the brain in a vat. So it seems that there is no way to know for sure that I am not in a matrix.

Let us call the hypothesis that I am in a matrix and have always been in a matrix the *Matrix Hypothesis*. Equivalently, the Matrix Hypothesis says that I am envatted and have always been envatted. This is not quite equivalent to the hypothesis that I am in the Matrix, as the Matrix is just one specific version of a matrix. For now, I will ignore some complications that are specific to the Matrix in the movie, such as the fact that people sometimes travel back and forth between the Matrix and the external world. These issues aside, we can think of the Matrix Hypothesis informally as saying that I am in the same sort of situation as people who have always been in the Matrix.

The Matrix Hypothesis is one that we should take seriously. As Nick Bostrom has suggested, it is not out of the question that in the history of the universe, technology will evolve that will allow beings to create computer simulations of entire worlds. There may well be vast numbers of such computer simulations, compared to just one real world. If so, there may well be many more beings who are in a matrix than beings who are not. Given all this, one might even infer that it is more likely that we are in a matrix than that we are not. Whether this is right or not, it certainly seems that we cannot be *certain* that we are not in a matrix.

Serious consequences seem to follow. My envatted counterpart seems to be massively deluded. It thinks it is in Tucson; it thinks it is sitting at a desk writing an article; it thinks it has a body. On the face of it, all of these beliefs are false. Likewise, it seems that if *I* am envatted, my own corresponding beliefs are false. If I am envatted, I am not really in Tucson; I am not really sitting at a desk; and I may not even have a body. So if I don't know that I am not envatted, then I don't know that I am in Tucson; I don't know that I am sitting at a desk; and I don't know that I have a body.

The Matrix Hypothesis threatens to undercut almost everything I know. It seems to be a *skeptical hypothesis*: a hypothesis that I cannot rule out, and one that would falsify most of my beliefs if it were true. Where there is a skeptical hypothesis, it looks like none of these beliefs count as genuine knowledge. Of course the beliefs *might* be true—I might be lucky, and not be envatted—but I can't rule out the possibility that they are false. So a skeptical hypothesis leads to *skepticism* about these beliefs: I believe these things, but I do not know them.

To sum up the reasoning: I don't know that I'm not in a matrix. If I'm in a matrix, I'm probably not in Tucson. So if I don't know that I'm not in a matrix, then I don't know that I'm in Tucson. The same goes for almost everything else I think I know about the external world.

# 2 Envatment Reconsidered

This is a standard way of thinking about the vat scenario. It seems that this view is also endorsed by the people who created *The Matrix*. On the DVD case for the movie, one sees the following:

Perception: Our day-in, day-out world is real.

Reality: That world is a hoax, an elaborate deception spun by all-powerful machines that control us. Whoa.

I think this view is not quite right. I think that even if I am in a matrix, my world is perfectly real. A brain in a vat is not massively deluded (at least if it has always been in the vat). Neo does not have massively false beliefs about the external world. Instead, envatted beings have largely *correct* beliefs about their world. If so, the Matrix Hypothesis is not a skeptical hypothesis, and its possibility does not undercut everything that I think I know.

Philosophers have held this sort of view before. The eighteenth-century philosopher George Berkeley held, in effect, that appearance is reality. (Recall Morpheus: "What is 'real'? How do you define 'real'? If you're talking about what you can feel, what you can smell, what you can taste and see, then real is simply electrical signals interpreted by your brain.") If this is right, then the world perceived by envatted beings is perfectly real: these beings experience all the right appearances, and appearance is reality. So on this view, even envatted beings have true beliefs about the world.

I have recently found myself embracing a similar conclusion, though for quite different reasons. I don't find the view that appearance is reality plausible, so I don't endorse Berkeley's reasoning. And until recently, it has seemed quite obvious to me that brains in vats would have massively false beliefs. But I now think there is a line of reasoning that shows that this is wrong.

I still think I cannot rule out the hypothesis that I am in a matrix. But I think that even if I am in a matrix, I am still in Tucson; I am still sitting at my desk; and so on. So the hypothesis that I am in a matrix is not a skeptical hypothesis. The same goes for Neo. At the beginning of the film, if he thinks "I have hair", he is correct. If he thinks "It is sunny outside", he is correct. And the same goes, of course, for the original brain in a vat. When it thinks, "I have a body", it is correct. When it thinks, "I am walking", it is correct.

This view may seem counterintuitive at first. Initially, it seemed quite counterintuitive to me. So I'll now present the line of reasoning that has convinced me that it is correct.

# **3** The Metaphysical Hypothesis

I will argue that the hypothesis that I am envatted is not a skeptical hypothesis, but a *metaphysical hypothesis*. That is, it is a hypothesis about the underlying nature of reality.

Where physics is concerned with the microscopic processes that underlie macroscopic reality, metaphysics is concerned with the fundamental nature of reality. A metaphysical hypothesis might make a claim about the reality that underlies physics itself. Alternatively, it might say something about the nature of our minds, or the creation of our world.

I think that the Matrix Hypothesis should be regarded as a metaphysical hypothesis with all three of these elements. It makes a claim about the reality underlying physics, about the nature of our minds, and about the creation of the world.

In particular, I think the Matrix Hypothesis is equivalent to a version of the following three-part Metaphysical Hypothesis. First, physical processes are fundamentally computational. Second, our cognitive systems are separate from physical processes, but interact with these processes. Third, physical reality was created by beings outside physical space-time.

Importantly, nothing about this Metaphysical Hypothesis is skeptical. The Metaphysical Hypothesis here tells us about the processes underlying our ordinary reality, but it does not entail that this reality does not exist. We still have bodies, and there are still chairs and tables: it's just that their fundamental nature is a bit different from what we may have thought. In this manner, the Metaphysical Hypothesis is analogous to a physical hypothesis, such as one involving quantum mechanics. Both the physical hypothesis and the Metaphysical Hypothesis tell us about the processes underlying chairs. They do not entail that there are no chairs. Rather, they tell us what chairs are really like.

I will make the case by introducing each of the three parts of the Metaphysical Hypothesis separately. I will suggest that each of them is coherent, and cannot be conclusively ruled out. And I will suggest that none of them is a skeptical hypothesis: even if they are true, most of our ordinary beliefs are still correct. The same goes for a combination of all three hypotheses. I will then argue that the Matrix Hypothesis is equivalent to this combination.

## (1) The Creation Hypothesis

The **Creation Hypothesis** says: Physical space-time and its contents were created by beings outside physical space-time.



This is a familiar hypothesis. A version of it is believed by many people in our society, and perhaps by the majority of the people in the world. If one believes that God created the world, and if one believes that God is outside physical space-time, then one believes the Creation Hypothesis. One needn't believe in God to believe the Creation Hypothesis, though. Perhaps our world was created by a relatively ordinary being in the "next universe up", using the latest world-making technology in that universe. If so, the Creation Hypothesis is true. I don't know whether the Creation Hypothesis is true. But I don't know for certain that it is false. The hypothesis is clearly coherent, and I cannot conclusively rule it out.

The Creation Hypothesis is not a skeptical hypothesis. Even if it is true, most of my ordinary beliefs are still true. I still have hands; I am still in Tucson; and so on. Perhaps a few of my beliefs will turn out to be false: if I am an atheist, for example, or if I believe all reality started with the Big Bang. But most of my everyday beliefs about the external world will remain intact.

#### (2) The Computational Hypothesis

The **Computational Hypothesis** says: Microphysical processes throughout space-time are constituted by underlying computational processes.



The Computational Hypothesis says that physics as we know it is not the fundamental level of reality. Just as chemical processes underlie biological processes, and microphysical processes underlie chemical processes, something underlies microphysical processes. Underneath the level of quarks, electrons, and photons is a further level: the level of bits. These bits are governed by a computational algorithm, which at a higher level produces the processes that we think of as fundamental particles, forces, and so on.

The Computational Hypothesis is not as widely believed as the Creation Hypothesis, but some people take it seriously. Most famously, Edward Fredkin has postulated that the universe is at bottom some sort of computer. More recently, Stephen Wolfram has taken up the idea in his book *A New Kind of Science*, suggesting that at the fundamental level, physical reality may be a sort of cellular automata, with interacting bits governed by simple rules. And some physicists have looked into the possibility that the laws of physics might be formulated computationally, or could be seen as the consequence of certain computational principles.

One might worry that pure bits could not be the fundamental level of reality: a bit is just a zero or a one, and reality can't really be zeroes and ones. Or perhaps a bit is just a "pure difference" between two basic states, and there can't be a reality made up of pure differences. Rather, bits always have to be implemented by more basic states, such as voltages in a normal computer.

I don't know whether this objection is right. I don't think it's completely out of the question that there could be a universe of pure bits. But this doesn't matter for present purposes. We can suppose that the computational level is itself constituted by an even more fundamental level, at which the computational processes are implemented. It doesn't matter for present purposes what that more fundamental level is. All that matters is that microphysical processes are constituted by computational processes, which are themselves constituted by more basic processes. From now on I will regard the Computational Hypothesis as saying this.

I don't know whether the Computational Hypothesis is correct. But again, I don't know that it is false. The hypothesis is coherent, if speculative, and I cannot conclusively rule it out.

The Computational Hypothesis is not a skeptical hypothesis. If it is true, there are still electrons and protons. On this picture, electrons and protons will be analogous to molecules: they are made up of something more basic, but they still exist. Similarly, if the Computational Hypothesis is true, there are still tables and chairs, and macroscopic reality still exists. It just turns out that their fundamental reality is a little different from what we thought.

The situation here is analogous to quantum mechanics or relativity. These may lead us to revise a few metaphysical beliefs about the external world: that the world is made of classical particles, or that there is absolute time. But most of our ordinary beliefs are left intact. Likewise, accepting the Computational Hypothesis may lead us to revise a few metaphysical beliefs: that electrons and protons are fundamental, for example. But most of our ordinary beliefs are unaffected.

#### (3) The Mind-Body Hypothesis

The **Mind-Body Hypothesis** says: My mind is (and has always been) constituted by processes outside physical space-time, and receives its perceptual inputs from and sends its outputs to processes in physical space-time.



The Mind-Body Hypothesis is also quite familiar and quite widely believed. Descartes believed something like this: on his view, we have nonphysical minds that interact with our physical bodies. The hypothesis is less widely believed today than in Descartes's time, but there are still many people who accept the Mind-Body Hypothesis.

Whether or not the Mind-Body Hypothesis is true, it is certainly coherent. Even if contemporary science tends to suggest that the hypothesis is false, we cannot rule it out conclusively.

The Mind-Body Hypothesis is not a skeptical hypothesis. Even if my mind is outside physical space-time, I still have a body; I am still in Tucson; and so on. At most, accepting this hypothesis would make us revise a few metaphysical beliefs about our minds. Our ordinary beliefs about external reality will remain largely intact.

## (4) The Metaphysical Hypothesis

We can now put these hypotheses together. First we can consider the Combination Hypothesis, which combines all three. It says that physical space-time and its contents were created by beings outside physical space-time, that microphysical processes are constituted by computational processes, and that our minds are outside physical space-time but interact with it.

As with the hypotheses taken individually, the Combination Hypothesis is coherent, and we cannot conclusively rule it out. And like the hypotheses taken individually, it is not a skeptical hypothesis. Accepting it might lead us to revise a few of our beliefs, but it would leave most of them intact.



Finally, we can consider the Metaphysical Hypothesis (with a capital M). Like the Combination Hypothesis, this combines the Creation Hypothesis, the Computational Hypothesis, and the Mind-Body Hypothesis. It also adds the following more specific claim: the computational processes underlying physical space-time were designed by the creators as a computer simulation of a world. (It may also be useful to think of the Metaphysical Hypothesis as saying that the computational processes constituting physical space-time are part of a broader domain, and that the creators and my cognitive system are also located within this domain. This addition is not strictly necessary for what follows, but it matches up with the most common way of thinking about the Matrix Hypothesis.)

The Metaphysical Hypothesis is a slightly more specific version of the Combination Hypothesis, in that it specifies some relations among the various parts of the hypothesis. Again, the Metaphysical Hypothesis is a coherent hypothesis, and we cannot conclusively rule it out. And again, it is not a skeptical hypothesis. Even if we accept it, most of our ordinary beliefs about the external world will be left intact.

# 4 The Matrix Hypothesis as a Metaphysical Hypothesis

Recall that the Matrix Hypothesis says: I have (and have always had) a cognitive system that receives its inputs from and sends its outputs to an artificially-designed computer simulation of a world.

I will argue that the Matrix Hypothesis is equivalent to the Metaphysical Hypothesis, in the following sense: if I accept the Metaphysical Hypothesis, I should accept the Matrix Hypothesis, and if I accept the Matrix Hypothesis, I should accept the Metaphysical Hypothesis. That is, the two hypotheses *imply* each other, where this means that if I accept one, I should accept the other.

Take the first direction first, from the Metaphysical Hypothesis to the Matrix Hypothesis. The Mind-Body Hypothesis implies that I have (and have always had) an isolated cognitive system which receives its inputs from and sends its outputs to processes in physical spacetime. In conjunction with the Computational Hypothesis, this implies that my cognitive system receives inputs from and sends outputs to the computational processes that constitute physical space-time. The Creation Hypothesis (along with the rest of the Metaphysical Hypothesis) implies that these processes were artificially designed to simulate a world. It follows that I have (and have always had) an isolated cognitive system that receives its inputs from and sends its outputs to an artificially designed computer simulation of a world. This is just the Matrix Hypothesis. So the Metaphysical Hypothesis implies the Matrix Hypothesis.

The other direction is closely related. To put it informally: if I accept the Matrix Hypothesis, I accept that what underlies apparent reality is just as the Metaphysical Hypothesis specifies. There is a domain containing my cognitive system, which is causally interacting with a computer simulation of physical space-time, which was created by other beings in that domain. This is just what has to obtain in order for the Metaphysical Hypothesis to obtain. If one accepts this, one should accept the Creation Hypothesis, the Computational Hypothesis, the Mind-Body Hypothesis, and the relevant relations among these.

This may be a little clearer through a picture. Here is the shape of the world according to the Matrix Hypothesis.



At the fundamental level, this picture of the shape of the world is exactly the same as the picture of the Metaphysical Hypothesis given above. So if one accepts that the world is as it is according to the Matrix Hypothesis, one should accept that it is as it is according to the Metaphysical Hypothesis.

One might make various objections. For example, one might object that the Matrix Hypothesis implies that a computer simulation of physical processes exists, but (unlike the Metaphysical Hypothesis) it does not imply that the physical processes themselves exist. I will discuss this objection in section 6, and other objections in section 7. For now, though, I take it that there is a strong case that the Matrix Hypothesis implies the Metaphysical Hypothesis, and vice versa.

# 5 Life in the Matrix

If this is right, it follows that the Matrix Hypothesis is not a skeptical hypothesis. If I accept it, I should not infer that the external world does not exist, nor that I have no body, nor that there are no tables and chairs, nor that I am not in Tucson. Rather, I should infer that the physical world is constituted by computations beneath the microphysical level. There are still tables, chairs, and bodies: these are made up fundamentally of bits and of whatever constitutes these bits. This world was created by other beings, but is still perfectly real. My mind is separate from physical processes, and interacts with them. My mind may not have been created by these beings, and it may not be made up of bits, but it still interacts with these bits.

The result is a complex picture of the fundamental nature of reality. The picture is strange and surprising, perhaps, but it is a picture of a full-blooded external world. If we are in a matrix, this is simply the way that the world is.

We can think of the Matrix Hypothesis as a creation myth for the information age. If it is correct, then the physical world was created, but necessarily by gods. Underlying the physical world is a giant computation, and creators created this world by implementing this computation. And our minds lie outside this physical structure, with an independent nature that interacts with this structure.

Many of the same issues that arise with standard creation myths arise here. When was the world created? Strictly speaking, it was not created within *our* time at all. When did history begin? The creators might have started the simulation in 4004 BC (or in 1999) with the fossil record intact, but it would have been much easier for them to start the simulation at the Big Bang and let things run their course from there. (In *The Matrix*, of course, the creators are machines. This gives an interesting twist on common theological readings of the movie. It is often held that Neo is the Christ figure in the movie, with Morpheus corresponding to John the Baptist, Cypher to Judas Iscariot, and so on. But on the reading I have given, the gods of *The Matrix* are the machines. Who, then, is the Christ figure? Agent Smith, of course! After all, he is the gods' offspring, sent down to save the Matrix-world from those who wish to destroy it. And in the second movie, he is even resurrected.)

Many of the same issues that arise on the standard Mind-Body Hypothesis also arise here. When do our nonphysical minds start to exist? It depends on just when new envatted cognitive systems are attached to the simulation (perhaps at the time of conception within the matrix, or perhaps at the time of birth?). Is there life after death? It depends on just what happens to the envatted systems once their simulated bodies die. How do mind and body interact? By causal links that are outside physical space and time.

Even if we are not in a matrix, we can extend a version of this reasoning to other beings who are in a matrix. If they discover their situation, and come to accept that they are in a matrix, they should not reject their ordinary beliefs about the external world. At most, they should come to revise their beliefs about the underlying nature of their world: they should come to accept that external objects are made of bits, and so on. These beings are not massively deluded: most of their ordinary beliefs about their world are correct.

There are a few qualifications here. One may worry about beliefs about other people's minds. I believe that my friends are conscious. If I am in a matrix, is this correct? In the Matrix depicted in the movie, these beliefs are mostly fine. This is a multi-vat matrix: for each of my perceived friends, there is an envatted being in the external reality, who is

presumably conscious like me. The exception might be beings such as Agent Smith, who are not envatted but are entirely computational. Whether these beings are conscious depends on whether computation is enough for consciousness. I will remain neutral on that issue here. We could circumvent this issue by building into the Matrix Hypothesis the requirement that all of the beings we perceive are envatted. But even if we do not build in this requirement, we are not much worse off than in the actual world, where there is a legitimate issue about whether other beings are conscious, quite independent of whether we are in a matrix.

One might also worry about beliefs about the distant past, and about the far future. These will be unthreatened as long as the computer simulation covers all of space-time, from the Big Bang until the end of the universe. This is built into the Metaphysical Hypothesis, and we can stipulate that it is built into the Matrix Hypothesis too, by requiring that the computer simulation be a simulation of an entire world. There may be other simulations that start in the recent past (perhaps the Matrix in the movie is like this), and there may be others that only last for a short while. In these cases, the envatted beings will have false beliefs about the past or the future in their worlds. But as long as the simulation covers the lifespan of these beings, it is plausible that they will have mostly correct beliefs about the current state of their environment.

There may be some respects in which the beings in a matrix are deceived. It may be that the creators of the matrix control and interfere with much of what happens in the simulated world. (The Matrix in the movie may be like this, though the extent of the creators' control is not quite clear.) If so, then these beings may have much less control over what happens than they think. But the same goes if there is an interfering god in a nonmatrix world. And the Matrix Hypothesis does not imply that the creators interfere with the world, though it leaves the possibility open. At worst, the Matrix Hypothesis is no more skeptical in this respect than is the Creation Hypothesis in a nonmatrix world.

The inhabitants of a matrix may also be deceived in that reality is much bigger than they think. They might think their physical universe is all there is, when in fact there is much more in the world, including beings and objects that they can never possibly see. But again, this sort of worry can arise equally in a nonmatrix world. For example, cosmologists seriously entertain the hypothesis that our universe may stem from a black hole in the "next universe up", and that in reality there may be a whole tree of universes. If so, the world is also much bigger than we think, and there may be beings and objects that we can never possibly see. But either way, the world that we see is perfectly real.

Importantly, none of these sources of skepticism—about other minds, the past and the future, our control over the world, and the extent of the world—casts doubt on our belief in

the reality of the world that we perceive. None of them leads us to doubt the existence of external objects such as tables and chairs, in the way that the vat hypothesis is supposed to do. And none of these worries is especially tied to the matrix scenario. One can raise doubts about whether other minds exist, whether the past and the future exist, and whether we have control over our world quite independently of whether we are in a matrix. If this is right, then the Matrix Hypothesis does not raise the distinctive skeptical issues that it is often taken to raise.

I suggested before that it is not out of the question that we really are in a matrix. One might have thought that this would be a worrying conclusion. But if I am right, it is not nearly as worrying as one might have thought. Even if we are in such a matrix, our world is no less real than we thought it was. It just has a surprising fundamental nature.

# 6 Objection: Simulation is not Reality

(This slightly technical section can be skipped without too much loss.)

A common line of objection is that a simulation is not the same as reality. The Matrix Hypothesis implies only that a simulation of physical processes exists. By contrast, the Metaphysical Hypothesis implies that physical processes really exist (they are explicitly mentioned in the Computational Hypothesis and elsewhere). If so, then the Matrix Hypothesis cannot imply the Metaphysical Hypothesis. On this view, if I am in a matrix, then physical processes do not really exist.

In response: My argument does not require the general assumption that simulation is the same as reality. The argument works quite differently. But the objection helps us to flesh out the informal argument that the Matrix Hypothesis implies the Metaphysical Hypothesis.

Because the Computational Hypothesis is coherent, it is clearly *possible* that a computational level underlies real physical processes, and it is possible that the computations here are implemented by further processes in turn. So there is *some* sort of computational system that could yield reality here. But here, the objector will hold that not all computational systems are created equal. To say that some computational systems will yield real physical processes in this role is not to say that they all do. Perhaps some of them are merely simulations. If so, then the Matrix Hypothesis may not yield reality.

To rebut this objection, we can appeal to two principles. First principle: any abstract computation that could be used to simulate physical space-time is such that it *could* turn out to underlie real physical processes. Second principle: given an abstract computation that *could* underlie physical processes, the precise way in which it is implemented is irrelevant to

whether it *does* underlie physical processes. In particular, the fact that the implementation was designed as a simulation is irrelevant. The conclusion then follows directly.

On the first principle: let us think of abstract computations in purely formal terms, abstracting away from their manner of implementation. For an abstract computation to qualify as a simulation of physical reality, it must have computational elements that correspond to every particle in reality (likewise for fields, waves, or whatever is fundamental), dynamically evolving in a way that corresponds to each particle's evolution. But then, it is guaranteed that the computation will have a rich enough causal structure that it *could* in principle underlie physics in our world. Any computation will do, as long as it has enough detail to correspond to the fine details of physical processes.

On the second principle: given an abstract computation that could underlie physical reality, it does not matter how the computation is implemented. We can imagine discovering that some computational level underlies the level of atoms and electrons. Once we have discovered this, it is possible that this computational level is implemented by more basic processes. There are many hypotheses about what the underlying processes could be, but none of them is especially privileged, and none of them would lead us to reject the hypothesis that the computational level constitutes physical processes. That is, the Computational Hypothesis is *implementation-independent*: as long as we have the right sort of abstract computation, the manner of implementation does not matter.

In particular, it is irrelevant whether or not these implementing processes were artificially created, and it is irrelevant whether they were intended as a simulation. What matters is the intrinsic nature of the processes, not their origin. And what matters about this intrinsic nature is simply that they are arranged in such a way as to implement the right sort of computation. If so, the fact that the implementation originated as a simulation is irrelevant to whether it can constitute physical reality.

There is one further constraint on the implementing processes: they must be connected to our experiences in the right sort of way. That is, when we have an experience of an object, the processes underlying the simulation of that object must be causally connected in the right sort of way to our experiences. If this is not the case, then there will be no reason to think that these computational processes underlie the physical processes that we perceive. If there is an isolated computer simulation to which nobody is connected in this way, we should say that it is simply a simulation. But an appropriate hook-up to our perceptual experiences is built into the Matrix Hypothesis, on the most natural understanding of that hypothesis. So the Matrix Hypothesis has no problems here. Overall, then, we have seen that computational processes *could* underlie physical reality, that any abstract computation that qualifies as a simulation of physical reality could play this role, and that any implementation of this computation could constitute physical reality, as long as it is hooked up to our experiences in the relevant way. The Matrix Hypothesis guarantees that we have an abstract computation of the right sort, and it guarantees that it is hooked up to our experiences in the relevant ways. So the Matrix Hypothesis implies that the Computational Hypothesis is correct, and that the computer simulation constitutes genuine physical processes.

# 7 Other Objections

When we look at a brain in a vat from the outside, it is hard to avoid the sense that it is deluded. This sense manifests itself in a number of related objections. These are not direct objections to the argument above, but they are objections to its conclusion.

*Objection 1:* A brain in a vat may think it is outside walking in the sun, when in fact it is alone in a dark room. Surely, it is deluded.

*Response:* The *brain* is alone in a dark room. But this does not imply that the *person* is alone in a dark room. By analogy, just say

Descartes is right that we have disembodied minds outside space-time, made of ectoplasm. When I think, "I am outside in the sun", an angel might look at my ectoplasmic mind and note that in fact it is not exposed to any sun at all. Does it follow that my thought is incorrect? Presumably not: I can be outside in the sun, even if my ectoplasmic mind is not. The angel would be wrong to infer that I have an incorrect belief. Likewise, we should not infer that the envatted being has an incorrect belief. At least, it is no more deluded than a Cartesian mind.

The moral is that the immediate surroundings of our minds may well be irrelevant to the truth of most of our beliefs. What matters is the processes that our minds are connected to, by perceptual inputs and motor outputs. Once we recognize this, the objection falls away.

*Objection 2:* An envatted being may believe that it is in Tucson, when in fact it is in New York, and has never been anywhere near Tucson. Surely, this belief is deluded.

*Response:* The envatted being's concept of 'Tucson' does not refer to what we call Tucson. Rather, it refers to something else entirely: call this 'Tucson\*' or 'virtual Tucson'. We might think of this as a virtual location (more on this in a moment). When the being says to itself, "I am in Tucson", it really is thinking that it is in Tucson\*, and it may well be in



Tucson\*. Because Tucson is not Tucson\*, the fact that the being has never been in Tucson is irrelevant to whether its belief is true.

A rough analogy: I look at my colleague Terry and think, "That's Terry". Elsewhere in the world, a duplicate of me looks at a duplicate of Terry. It thinks, "That's Terry", but it is not looking at the real Terry. Is its belief false? It seems not: my duplicate's "Terry" concept refers not to Terry, but to his duplicate Terry\*. My duplicate really is looking at Terry\*, so its belief is true. The same sort of thing is happening in the case above.

*Objection 3:* Before he leaves the Matrix, Neo believes that he has hair. But in reality he has no hair (the body in the vat is bald). Surely this belief is deluded.

*Response:* This case is like the last one. Neo's concept of 'hair' does not refer to real hair, but to something else that we might call hair\* (virtual hair). So the fact that Neo does not have real hair is irrelevant to whether his belief is true. Neo really does have virtual hair, so he is correct. Likewise, when a child in the movie tells Neo, "There is no spoon", his concept refers to a virtual spoon, and there really is a virtual spoon. So the child is wrong.

*Objection 4:* What *sort* of objects does an envatted being refer to? What *is* virtual hair, virtual Tucson, and so on?

*Response:* These are all entities constituted by computational processes. If I am envatted, then the objects to which I refer (hair, Tucson, and so on) are all made of bits. And if another being is envatted, the objects that it refers to (hair\*, Tucson\*, and so on) are likewise made of bits. If the envatted being is hooked up to a simulation in my computer, then the objects to which it refers are constituted by patterns of bits inside my computer. We might call these things *virtual objects*. Virtual hands are not hands (assuming I am not envatted), but they exist inside the computer all the same. Virtual Tucson is not Tucson, but it exists inside the computer all the same.

*Objection 5:* You just said that virtual hands are not real hands. Does this mean that if we are in a matrix, we don't have real hands?

*Response:* No. If we are *not* in a matrix, but other beings are, we should say that their term *hand* refers to virtual hands, but our term does not. So in this case, our hands aren't virtual hands. But if we *are* in a matrix, then our term 'hand' refers to something that's made of bits: virtual hands, or at least something that would be regarded as virtual hands by people in the next world up. That is, if we *are* in a matrix, real hands are made of bits. Things look quite different, and our words refer to different things, depending on whether our perspective is from inside or outside the matrix.

This sort of perspective shift is common in thinking about the matrix scenario. From the first-person perspective, we suppose that *we* are in a matrix. Here, real things in our world are

made of bits, though the "next world up" might not be made of bits. From the third-person perspective we suppose that someone *else* is in a matrix but we are not. Here, real things in our world are not made of bits, but the "next world down" is made of bits. On the first way of doing things, our words refer to computational entities. On the second way of doing things, the envatted beings' words refer to computational entities, but our words do not.

*Objection 6:* Just which pattern of bits is a given virtual object? Surely it will be impossible to pick out a precise set.

*Response:* This question is like asking: just which part of the quantum wave function is this chair, or is the University of Arizona? These objects are all ultimately constituted by an underlying quantum wave function, but there may be no precise part of the micro-level wave function that we can say "is" the chair or the university. The chair and the university exist at a higher level. Likewise, if we are envatted, there may be no precise set of bits in the micro-level computational process that is the chair or the university. These exist at a higher level. And if someone else is envatted, there may be no precise sets of bits in the computer simulation that "are" the objects to which they refer. But just as a chair exists without being any precise part of the wave function, a virtual chair may exist without being any precise set of bits.

*Objection 7:* An envatted being thinks it performs actions, and it thinks it has friends. Are these beliefs correct?

*Response:* One might try to say that the being performs actions\* and that it has friends\*. But for various reasons I think it is not plausible that words like 'action' and 'friend' can shift their meanings as easily as words like like 'Tucson' and 'hair'. Instead, I think one can say truthfully (in our own language) that the envatted being performs actions, and that it has friends. To be sure, it performs actions in *its* environment, and its environment is not our environment but the virtual environment. And its friends likewise inhabit the virtual environment (assuming that we have a multivat matrix, or that computation suffices for consciousness). But the envatted being is not incorrect in this respect.

*Objection 8:* Set these technical points aside. Surely, if we are in a matrix, the world is nothing like we think it is.

*Response:* I deny this. Even if we are in a matrix, there are still people, football games, and particles, arranged in space-time just as we think they are. It is just that the world has a *further* nature that goes beyond our initial conception. In particular, things in the world are realized computationally in a way that we might not have originally imagined. But this does not contradict any of our ordinary beliefs. At most, it will contradict a few of our more

abstract metaphysical beliefs. But exactly the same goes for quantum mechanics, relativity theory, and so on.

If we are in a matrix, we may not have many false beliefs, but there is much knowledge that we lack. For example, we do not know that the universe is realized computationally. But this is just what one should expect. Even if we are not in a matrix, there may well be much about the fundamental nature of reality that we do not know. We are not omniscient creatures, and our knowledge of the world is at best partial. This is simply the condition of a creature living in a world.

## 8 Other Skeptical Hypothesis

The Matrix Hypothesis is one example of a traditional "skeptical" hypothesis, but it is not the only example. Other skeptical hypotheses are not quite as straightforward as the Matrix Hypothesis. Still, I think that for many of them, a similar line of reasoning applies. In particular, one can argue that most of these are not global skeptical hypotheses: that is, their truth would not undercut all of our empirical beliefs about the physical world. At worst, most of them are *partial* skeptical hypotheses, undercutting some of our empirical beliefs, but leaving many other beliefs intact.

**New Matrix Hypothesis:** I was recently created, along with all of my memories, and was put in a newly created matrix.

What if both the matrix and I have existed for only a short time? This hypothesis is a computational version of Bertrand Russell's Recent Creation Hypothesis: the physical world was created only recently (with fossil record intact), and so was I (with memories intact). On that hypothesis, the external world that I perceive really exists, and most of my beliefs about its current state are plausibly true, but I have many false beliefs about the past. I think the same should be said of the New Matrix Hypothesis. One can argue, along the lines presented earlier, that the New Matrix Hypothesis is equivalent to a combination of the Metaphysical Hypothesis with the Recent Creation Hypothesis. This combination is not a global skeptical hypothesis (though it is a partial skeptical hypothesis, where beliefs about the past are concerned). So the same goes for the New Matrix Hypothesis.

**Recent Matrix Hypothesis:** For most of my life I have not been envatted, but I was recently hooked up to a matrix.

If I was recently put in a matrix without realizing it, it seems that many of my beliefs about my current environment are false. Let's say that just yesterday someone put me into a simulation, in which I fly to Las Vegas and gamble at a casino. Then I may believe that I am in Las Vegas now, and that I am in a casino, but these beliefs are false: I am really in a laboratory in Tucson.

This result is quite different from the long-term matrix. The difference lies in the fact that my conception of external reality is anchored to the reality in which I have lived most of my life. If I have been envatted all of my life, my conception is anchored to the computationally constituted reality. But if I were just envatted yesterday, my conception is anchored to the external reality. So when I think that I am in Las Vegas, I am thinking that I am in the external Las Vegas, and this thought is false.

Still, this does not undercut all of my beliefs about the external world. I believe that I was born in Sydney, that there is water in the oceans, and so on, and all of these beliefs are correct. It is only my recently acquired beliefs, stemming from my perception of the simulated environment, that will be false. So this is only a partial skeptical hypothesis: its possibility casts doubt on a subset of our empirical beliefs, but it does not cast doubt on all of them.

Interestingly, the Recent Matrix and the New Matrix hypotheses give opposite results, despite their similar nature: the Recent Matrix Hypothesis yields true beliefs about the past but false beliefs about the present, while the New Matrix Hypothesis yields false beliefs about the past and true beliefs about the present. The differences are tied to the fact that in the Recent Matrix Hypothesis, I really have a past existence for my beliefs to be about, and that past reality has played a role in anchoring the contents of my thoughts, which has no parallel under the New Matrix Hypothesis.

**Local Matrix Hypothesis:** I am hooked up to a computer simulation of a fixed local environment in a world.

On one way of doing this, a computer simulates a small fixed environment in a world, and the subjects in the simulation encounter some sort of barrier when they try to leave that area. For example, in the movie *The Thirteenth Floor*, just California is simulated, and when the subject tries to drive to Nevada, a road sign says "Closed for Repair" (with faint green electronic mountains in the distance!). Of course this is not the best way to create a matrix, as subjects are likely to discover the limits to their world.

This hypothesis is analogous to a Local Creation Hypothesis, on which creators just created a local part of the physical world. Under this hypothesis, we will have true beliefs about nearby matters, but false beliefs about matters farther from home. By the usual sort of reasoning, the Local Matrix Hypothesis can be seen as a combination of the Metaphysical Hypothesis with the Local Creation Hypothesis. So we should say the same thing about this.

**Extendible Local Matrix Hypothesis:** I am hooked up to a computer simulation of a local environment in a world, which is extended when necessary depending on my movements.

This hypothesis avoids the obvious difficulties with a fixed local matrix. Here, the creators simulate a local environment and extend it when necessary. For example, they might right now be concentrating on simulating a room in my house in Tucson. If I walk into another room, or fly to another city, they will simulate those. Of course, they need to make sure that when I go to these places, they match my memories and beliefs reasonably well, with allowance for evolution in the meantime. The same goes for when I encounter familiar people, or people I have only heard about. Presumably, the simulators keep up a database of the information about the world that has been settled so far, updating this information whenever necessary as time goes along, and making up new details when they need them.

This sort of simulation is quite unlike simulation in an ordinary matrix. In a matrix, the whole world is simulated at once. There are high start-up costs, but once the simulation is up and running, it will take care of itself. By contrast, the extendible local matrix involves "just-in-time" simulation. This has much lower start-up costs, but it requires much more work and creativity as the simulation evolves.

This hypothesis is analogous to an Extendible Local Creation Hypothesis about ordinary reality, under which creators create just a local physical environment and extend it when necessary. Here, external reality exists and many local beliefs are true, but again beliefs about matters farther from home are false. If we combine that hypothesis with the Metaphysical Hypothesis, the result is the Extendible Local Matrix Hypothesis. So if we are in an extendible local matrix, external reality still exists, but there is not as much of it as we thought. Of course if I travel in the right direction, more of it may come into existence.

The situation is reminiscent of the film *The Truman Show*. Truman lives in an artificial environment made up of actors and props, which behave appropriately when he is around, but which may be completely different when he is absent. Truman has many true beliefs about his current environment: there really are tables and chairs in front of him, and so on. But he is deeply mistaken about things outside his current environment, and farther from home.

It is common to think that while *The Truman Show* poses a disturbing skeptical scenario, *The Matrix* is much worse. But if I am right, things are reversed. If I am in a matrix, then most of my beliefs about the external world are true. If I am in something like *The Truman* 

*Show*, then a great number of my beliefs are false. On reflection, it seems to me that this is the right conclusion. If we were to discover that we were (and always had been) in a matrix, this would be surprising, but we would quickly get used to it. If we were to discover that we were (and always had been) in a televised "Truman Show", we might well go insane.

**Macroscopic Matrix Hypothesis:** I am hooked up to a computer simulation of macroscopic physical processes without microphysical detail.

One can imagine that, for ease of simulation, the makers of a matrix might not bother to simulate low-level physics. Instead, they might just represent macroscopic objects in the world and their properties: for example, that there is a table with such-and-such shape, position, and color, with a book on top of it with certain properties, and so on. They will need to make some effort to make sure that these objects behave in physically reasonable ways, and they will have to make special provisions for handling microphysical measurements, but one can imagine that at least a reasonable simulation could be created this way.

I think this hypothesis is analogous to a Macroscopic World Hypothesis: there are no microphysical processes, and instead macroscopic physical objects exist as fundamental objects in the world, with properties of shape, color, position, and so on. This is a coherent way that our world could be, and it is not a global skeptical hypothesis, though it may lead to false scientific beliefs about lower levels of reality. The Macroscopic Matrix Hypothesis can be seen as a combination of this hypothesis with a version of the Metaphysical Hypothesis. As such, it is not a global skeptical hypothesis either.

One can also combine the various hypotheses above in various ways, yielding hypotheses such as a New Local Macroscopic Matrix Hypothesis. For the usual reasons, all of these can be seen as analogs of corresponding hypotheses about the physical world. So all of them are compatible with the existence of physical reality, and none is a global skeptical hypothesis.

**God Hypothesis:** Physical reality is represented in the mind of God, and our own thoughts and perceptions depend on God's mind.

A hypothesis like this was put forward by George Berkeley as a view about how our world might really be. Berkeley intended this as a sort of metaphysical hypothesis about the nature of reality. Most other philosophers have differed from Berkeley in regarding this as a sort of skeptical hypothesis. If I am right, Berkeley is closer to the truth. The God Hypothesis can be seen as a version of the Matrix Hypothesis, on which the simulation of the world is implemented in the mind of God. If this is right, we should say that physical processes really exist: it's just that at the most fundamental level, they are constituted by processes in the mind of God. **Evil Genius Hypothesis:** I have a disembodied mind, and an evil genius is feeding me sensory inputs to give the appearance of an external world.

This is René Descartes's classical skeptical hypothesis. What should we say about it? This depends on just how the evil genius works. If the evil genius simulates an entire world in his head in order to determine what inputs I should receive, then we have a version of the God Hypothesis. Here we should say that physical reality exists and is constituted by processes within the mind of the evil genius. If the evil genius is simulating only a small part of the physical world, just enough to give me reasonably consistent inputs, then we have an analog of the Local Matrix Hypothesis (in either its fixed or flexible versions). Here we should say that just a local part of external reality exists. If the evil genius is not bothering to simulate the microphysical level, but just the macroscopic level, then we have an analog of the Macroscopic Matrix Hypothesis. Here we should say that local external macroscopic objects exist, but our beliefs about their microphysical nature are incorrect.

The Evil Genius Hypothesis is often taken to be a global skeptical hypothesis. But if the reasoning above is right, this is incorrect. Even if the Evil Genius Hypothesis is correct, some of the external reality that we apparently perceive really exists, though we may have some false beliefs about it, depending on details. It is just that this external reality has an underlying nature that is quite different from what we may have thought.

Dream Hypothesis: I am now and have always been dreaming.

Descartes raised the question: how do you know that you are not currently dreaming? Morpheus raises a similar question:

Have you ever had a dream, Neo, that you were so sure was real. What if you were unable to wake from that dream? How would you know the difference between the dream world and the real world?

The hypothesis that I am *currently* dreaming is analogous to a version of the Recent Matrix Hypothesis. I cannot rule it out conclusively, and if it is correct, then many of my beliefs about my current environment are incorrect. But presumably, I still have many true beliefs about the external world, anchored in the past.

What if I have always been dreaming? That is, what if all of my apparent perceptual inputs have been generated by my own cognitive system, without my realizing this? I think this case is analogous to the Evil Genius Hypothesis: it's just that the role of the "evil genius" is played by a part of my own cognitive system! If my dream-generating system simulates all of space-time, we have something like the original Matrix Hypothesis. If it models just my local environment, or just some macroscopic processes, we have analogs of the more local

versions of the Evil Genius Hypothesis. In any of these cases, we should say that the objects that I am currently perceiving really exist (although objects farther from home may not). It is just that some of them are constituted by my own cognitive processes.

**Chaos Hypothesis:** I do not receive inputs from anywhere in the world. Instead, I have random, uncaused experiences. Through a huge coincidence, they are exactly the sort of regular, structured experiences with which I am familiar.

The Chaos Hypothesis is an extraordinarily unlikely hypothesis, much more unlikely than anything considered above. But it is still one that could in principle obtain, even if it has minuscule probability. If I am chaotically envatted, do physical processes in the external world exist? I think we should say that they do not. My experiences of external objects are caused by nothing, and the set of experiences associated with my conception of a given object will have no common source. Indeed, my experiences are not caused by any reality external to them at all. So this is a genuine skeptical hypothesis: if accepted, it would cause us to reject most of our beliefs about the external world.

So far, the only clear case of a global skeptical hypothesis is the Chaos Hypothesis. Unlike the previous hypotheses, accepting this hypothesis would undercut all of our substantive beliefs about the external world. Where does the difference come from?

Arguably, what is crucial is that on the Chaos Hypothesis, there is no causal explanation of our experiences at all, and there is no explanation for the regularities in our experience. In all of the previous cases, there is some explanation for these regularities, though perhaps not the explanation that we expect. One might suggest that as long as a hypothesis involves *some* reasonable explanation for the regularities in our experience, then it will not be a global skeptical hypothesis.

If so, then if we are granted the assumption that there is some explanation for the regularities in our experience, then it is safe to say that some of our beliefs about the external world are correct. This is not much, but it is something.

# 9 Philosophical Notes

The material above was written to be accessible to a wide audience, so it deliberately omits technical philosophical details, connections to the literature, and so on. In this section I will remedy this omission. Readers without a background in philosophy may choose to skip or skim this section.

#### Note 1

Hilary Putnam (1981) has argued that the hypothesis that I am (and have always been) a brain in a vat can be ruled out a priori. In effect, this is because my word 'brain' refers to objects in my perceived world, and it cannot refer to objects in an "outer" world in which the vat would have to exist. For my hypothesis "I am a brain in a vat" to be true, I would have to be a brain of the sort that exists in the perceived world, but that cannot be the case. So the hypothesis must be false.

An analogy: I can arguably rule out the hypothesis that I am in the Matrix (capital M). My term 'the Matrix' refers to a specific system that I have seen in a movie in my perceived world. I could not be in that very system as the system exists within the world that I perceive. So my hypothesis "I am in the Matrix" must be false.

This conclusion about the Matrix seems reasonable, but there is a natural response. Perhaps this argument rules out the hypothesis that I am in the Matrix, but I cannot rule out the hypothesis that I am in a matrix, where a matrix is a generic term for a computer simulation of a world. The term 'Matrix' may be anchored to the specific system in the movie, but the generic term 'matrix' is not.

Likewise, it is arguable that I can rule out the hypothesis that I am a brain in a vat (if 'brain' is anchored to a specific sort of biological system in my perceived world). But I cannot rule out the hypothesis that I am envatted, where this simply says that I have a cognitive system that receives input from and sends outputs to a computer simulation of a world. The term 'envatted' (and the terms used in its definition) are generic terms, not anchored to specific systems in perceived reality. By using this slightly different language, we can restate the skeptical hypothesis in a way that is invulnerable to Putnam's reasoning.

More technically: Putnam's argument may work for 'brain' and 'Matrix' because one is a natural kind term and the other is a name. These terms are subject to "Twin Earth" thought-experiments (Putnam 1975), where duplicates can use corresponding terms with different referents. On Earth, Oscar's term 'water' refers to H<sub>2</sub>O, but on Twin Earth (which contains the superficially identical XYZ in its oceans and lakes), Twin Oscar's term 'water' refers to XYZ. Likewise, perhaps my term 'brain' refers to biological brains, while an envatted being's term 'brain' refers to virtual brains. If so, when an envatted being says, "I am a brain in a vat", it is not referring to its biological brain, and its claim is false.

But not all terms are subject to Twin Earth thought-experiments. In particular, *semantically neutral* terms are not (at least when used without semantic deference): such terms plausibly include 'philosopher', 'friend', and many others. Other such terms include 'matrix' and 'envatted', as defined earlier. If we work with hypotheses such as "I am in a

matrix" and "I am envatted", rather than "I am in the Matrix" or "I am a brain in a vat", then Putnam's argument does not apply. Even if a brain in a vat could not truly think, "I am a brain in a vat", it could truly think, "I am envatted". So I think that Putnam's line of reasoning is ultimately a red herring.

## Note 2

Despite this disagreement, my main conclusion is closely related to another suggestion of Putnam's. This is the suggestion that a brain in a vat may have true beliefs, because it will refer to chemical processes or processes inside a computer. However, I reach this conclusion by a quite different route. Putnam argues by an appeal to the causal theory of reference: thoughts refer to what they are causally connected to, and the thoughts of an envatted being are causally connected to processes in a computer. This argument is clearly inconclusive, as the causal theory of reference is so unconstrained. To say that a causal connection is required for reference is not to say what sort of causal connection suffices. There are many cases (like "phlogiston") where terms fail to refer despite rich causal connections. Intuitively, it is natural to think that the brain in a vat is a case like this, so an appeal to the causal theory of reference does not seem to help.

The argument I have given presupposes nothing about the theory of reference. Instead, it proceeds directly by considering first-order hypotheses about the world, the connections among these, and what we should say if they are true. In answering objections, I have made some claims about reference, and these claims are broadly compatible with a causal theory of reference. But importantly, these claims are very much consequences of the first-order argument rather than presuppositions of it. In general, I think that claims in the theory of reference are beholden to first-order judgments about cases, rather than vice versa.

#### Note 3

I use "skeptical hypothesis" in a certain technical sense. A skeptical hypothesis (relative to a belief that P) is a hypothesis such that (i) we cannot rule it out with certainty; and (ii) were we to accept it, we would reject the belief that P. A skeptical hypothesis with respect to a class of beliefs is one that is a skeptical hypothesis with respect to most or all of the beliefs in that class. A global skeptical hypothesis is a skeptical hypothesis with respect to all of our empirical beliefs.

The existence of a skeptical hypothesis (with respect to a belief) casts doubt on the relevant belief, in the following sense. Because we cannot rule out the hypothesis with certainty, and because the hypothesis implies the negation of these beliefs, it seems (given a

plausible closure principle about certainty) that our knowledge of these beliefs is not certain. If it is also the case that we do not *know* that the skeptical hypothesis does not obtain (as I think is the case for most of the hypotheses in this article), then it follows from an analogous closure principle that the beliefs in the class do not constitute knowledge.

Some use "skeptical hypothesis" in a broader sense, to apply to any hypothesis such that if it obtains, I do not know that P. (A hypothesis under which I have accidentally true beliefs is a skeptical hypothesis in this sense but not in the previous sense.) I have not argued here that the Matrix Hypothesis is not a skeptical hypothesis in this sense. I have argued that if the hypothesis obtains, our beliefs are true, but I have not argued that if it obtains, our beliefs constitute knowledge. Nevertheless, I am inclined to think that if we have knowledge in an ordinary, nonmatrix world, we would also have knowledge in a matrix.

## Note 4

What is the relevant class of beliefs? Of course there are some beliefs that even a noexternal-world skeptical hypothesis might not undercut: the belief that I exist, or the belief that 2 + 2 = 4, or the belief that there are no unicorns. Because of this, it is best to restrict attention to beliefs that (i) are about the external world, (ii) are not justifiable a priori, and (iii) make a positive claim about the world (they could not be true in an empty world). For the purposes of this chapter we can think of these beliefs as our "empirical beliefs". Claims about skeptical hypotheses undercutting beliefs should generally be understood as restricted to beliefs in this class.

#### Note 5

On the Computational Hypothesis: it is coherent to suppose that there is a computational level underneath physics, but it is not clear whether it is coherent to suppose that this level is fundamental. If it is, then we have a world of "pure bits". Such a world would be a world of pure differences: there would be two basic states that differ from one another, without this difference being a difference in some deeper nature. Whether one thinks this is coherent or not is connected to whether one thinks that all differences must be grounded in some basic intrinsic nature, on whether one thinks that all dispositions must have a categorical bases, and so on. For the purposes of this chapter, however, the issue can be set aside. Under the Matrix Hypothesis, the computation itself is *implemented* by processes in the world of the creator. As such, there will be a more basic level of intrinsic properties that serves as the basis for the differences between bits.

## Note 6

On the Mind-Body Hypothesis: it is interesting to note that the Matrix Hypothesis shows a concrete way in which Cartesian substance dualism might have turned out to be true. It is sometimes held that the idea of physical processes interacting with a nonphysical mind is not just implausible but incoherent. The Matrix Hypothesis suggests fairly straightforwardly that this is wrong. Under this hypothesis, our cognitive system involves processes quite distinct from the processes in the physical world, but there is a straightforward causal story about how they interact.

Some questions arise. For example, if the envatted cognitive system is producing a body's motor outputs, what role does the simulated brain play? Perhaps one could do without it, but this will cause all sorts of awkward results, not least when doctors in the matrix open the skull. It is more natural to think that the envatted brain and the simulated brain will always be in isomorphic states, receiving the same inputs and producing the same outputs. If the two systems start in isomorphic states and always receive the same inputs, then (setting aside indeterminism) they will always stay in isomorphic states. As a bonus, this may explain why death in the Matrix leads to death in the outer world!

Which of these actually controls the body? This depends on how things are set up. Things might be set up so the envatted system's outputs are not fed back to the simulation; in this case a version of epiphenomenalism will be true. Things might be set up so that motor impulses in the simulated body depend on the envatted system's outputs with the simulated brain's outputs being ignored; in this case a version of interactionism will be true. Interestingly, this last might be a version of interactionism that is compatible with causal closure of the physical. A third possibility is that the mechanism takes both sets of outputs into account (perhaps averaging the two?). This could yield a sort of redundancy in the causation. Perhaps the controllers of the matrix might even sometimes switch between the two. In any of these cases, as long as the two systems stay in isomorphic states, the behavioral results will be the same.

One might worry that there will be two conscious minds here, in a fashion reminiscent of Daniel Dennett's story "Where Am I?" This depends on whether computation in the matrix is enough to support a mind. If anti-computationalists about the mind (such as John Searle) are right, there will be just one mind. If computationalists about the mind are right, there may well be two synchronized minds (which then raises the question: if I am in a matrix, which of the two minds is mine?). The one-mind view is certainly closer to the ordinary conception of reality, but the two-mind view is not out of the question.

One bonus of the computationalist view is that it allows us to entertain the hypothesis that we are in a computer simulation *without* a separate cognitive system attached. Instead, the creators just run the simulation, including a simulation of brains, and minds emerge within it. This is presumably much easier for the creators, as it removes any worries tied to the creation and upkeep of the attached cognitive systems. Because of this, it seems quite plausible that there will be many simulations of this sort in the future, whereas it is unclear that there will be many of the more cumbersome Matrix-style simulations. (Because of this, Bostrom's argument that we may well be in a simulation applies more directly to this sort of simulation than to Matrix-style simulations.) The hypothesis that we are in this sort of computer simulation corresponds to a slimmed-down version of the Metaphysical Hypothesis, on which the Mind-Body Hypothesis is unnecessary. As before, this is a nonskeptical hypothesis: if we are in such a simulation (and if computationalism about the mind is true), then most of our beliefs about the external world are still correct.

There are also other possibilities. One intriguing possibility (discussed in Chalmers 1990) is suggested by contemporary work in artificial life, which involves relatively simple simulated environments and complex rules by which simulated creatures interact with these environments. Here the algorithms responsible for the creatures' "mental" processes are quite distinct from those governing the "physics" of the environment. In this sort of simulation, creatures will presumably never find underpinnings for their cognitive processes in their perceived world. If these creatures become scientists, they will be Cartesian dualists, holding (correctly) that their cognitive processes lie outside their physical world. It seems that this is another coherent way that Cartesian dualism might have turned out to be true.

#### Note 7

I have argued that the Matrix Hypothesis implies the Metaphysical Hypothesis and vice versa. Here, 'implies' is an epistemic relation: if one accepts the first, one should accept the second. I do not claim that the Matrix Hypothesis *entails* the Metaphysical Hypothesis, in the sense that in any counterfactual world in which the Matrix Hypothesis holds, the Metaphysical Hypothesis holds. That claim seems to be false. For example, there are counterfactual worlds in which physical space-time is created by nobody (so the Metaphysical Hypothesis is false), in which I am hooked up to an artificially-designed computer simulation located within physical space-time (so the Matrix Hypothesis is true). And if physics is not computational in the actual world, then physics in this world is not computational either. One might say that the two hypothesis are *a priori* equivalent, but not necessarily equivalent. (Of course the term 'physics' as used by my envatted self in the counterfactual world will refer to

something that is both computational and created. But 'physics' as used by my current nonenvatted self picks out the outer noncomputational physics of that world, not the computational processes.)

The difference arises from two different ways of considering the Matrix Hypothesis: as a hypothesis about what might actually be the case, or as a hypothesis about what might have been the case but is not. The first hypothesis is reflected in indicative conditionals: if I am actually in a matrix, then I have hands; atoms are made of bits; and the Metaphysical Hypothesis is true. The second version is reflected in subjunctive conditionals: if I had been in a matrix, I would not have had hands; atoms would not have been made of bits; and the Metaphysical Hypothesis would not have been true.

This is analogous to the different ways of thinking about Putnam's Twin Earth scenario, common in discussions of two-dimensional semantics. If I am actually in the XYZ-world, then XYZ is water, but if I had been in the XYZ-world, XYZ would not have been water (water would still have been H<sub>2</sub>O). On the first way of doing things, we consider a Twin Earth world as *actual*. On the second way of doing things, we consider a Twin Earth world as *counterfactual*. We can say that the Twin Earth world *verifies* that "water is XYZ", but that it *satisfies* "water is not XYZ", where verification and satisfaction correspond to considering something as actual and as counterfactual.

Likewise, we can say that a matrix world verifies the Metaphysical Hypothesis, but it does not satisfy the Metaphysical Hypothesis. The reason is that the Metaphysical Hypothesis makes claims about physics and the physical world. And what counts as 'physics' differs depending on whether the matrix world is considered as actual or counterfactual. If I am in a matrix, physics is computational. But if I *had been* in a matrix, physics would not have been computational (the matrix would have been computational, but the computer and my brain would all have been made from computation-independent physics). In this way, claims about physics and physical processes in a matrix world are analogous to claims about 'water' in the Twin Earth world.

#### Note 8

The responses to the first few objections in section 7 are clearly congenial to a causal account of reference. I said that the truth of an envatted being's thoughts depends not on its immediate environment but on what it is causally connected to, that is, on the computational processes to which it is hooked up. As noted earlier, I did not need to assume the causal theory of reference to get to this conclusion, but instead got there through a first-order argument. But once the conclusion is reached, there are many interesting points of contact.

For example, the idea that my term 'hair' refers to hair while my envatted counterpart's term refers to virtual hair has a familiar structure. It is structurally analogous to a Twin Earth case, in which Oscar (on Earth) refers to water (H2O) while his counterpart Twin Oscar (on Twin Earth) refers to twin water (XYZ). In both cases, the terms refer to what they are causally connected to. These natural-kind terms function by picking out a certain kind in the subject's environment, and the precise nature of that kind depends on the nature of the environment. Something similar applies to names for specific entities, such as 'Tucson'.

The behavior of these terms can be modelled using the two-dimensional semantic framework. As before, when we consider a Twin Earth world as actual, it verifies "water is XYZ", and when we consider it as counterfactual, it satisfies "water is not XYZ". Likewise, when we consider a matrix world as actual, it verifies "hair is made of bits", and when we consider it as counterfactual, it satisfies "hair is not made of bits".

The difference between considering something as actual or as counterfactual yields a perspective shift like the one in the response to objection 5. If the matrix world is considered as merely counterfactual, we should say that the beings in the matrix don't have hair (they only have virtual hair). But if the matrix world is considered as actual (that is, if we hypothetically accept that we are in a matrix), we should say that the beings in the matrix have hair, and that hair is itself a sort of virtual hair.

The Twin Earth analogy may suggest that the meanings of our terms such as 'hair' and the contents of our corresponding thoughts depend on our environment. But the twodimensional approach also suggests that there is an internal aspect of content that is shared between twins, and that does not depend on the environment. The *primary intension* of a sentence is true at a world if the world verifies the sentence, while its *secondary intension* is true at a world if the world satisfies the sentence. Then Oscar and Twin Oscar's sentences "water is wet" have different secondary intensions (roughly, true when H<sub>2</sub>O is wet or when XYZ is wet respectively), but they have the same primary intension (roughly, true at worlds where the watery-looking stuff is wet). Likewise, "I have hair" as used by me and my envatted counterpart has different secondary intensions (roughly, true in worlds where we have biological hair or computational hair respectively), but they have the same primary intensions (roughly, true in worlds where we have hair-looking stuff). The primary intensions of our thought and our language represent a significant shared dimension of content.

## Note 9

Why the different response to objection 7, on 'action' and 'friend'? We noted earlier (note 1) that not all terms function like 'water' and 'hair'. There are numerous *semantically* 

*neutral* terms that are not subject to Twin Earth thought-experiments: any two twins using these terms in different environments will use them with the same meaning (at least if they are using the terms without semantic deference). These terms arguably include 'and', 'friend', 'philosopher', 'action', 'experience', and 'envatted'. So while an envatted beings' term 'hand' or 'hair' or 'Tucson' may mean something different from our corresponding term, an envatted beings' term 'friend' or 'philosopher' or 'action' will arguably mean the same as ours.

It follows that if we are concerned with an envatted being's belief "I have friends", or "I perform actions", we cannot use the Twin Earth response. These beliefs will be true if and only if the envatted being has friends and performs actions. Fortunately, it seems quite reasonable to say that the envatted being *does* have friends (in its environment, not in ours), and that it does perform actions (in its environment, not in ours). The same goes for other semantically neutral terms: it is for precisely this class of expressions that this response is reasonable.

#### Note 10

What is the ontology of virtual objects? This is a hard question, but it is no harder than the question of the ontology of ordinary macroscopic objects in a quantum-mechanical world. The response to objection 6 suggests that in both cases, we should reject claims of token identity between microscopic and macroscopic levels. Tables are not identical to any object characterized purely in terms of quantum mechanics; likewise, virtual tables are not identical to any objects characterized purely in terms of bits. But nevertheless, facts about tables supervene on quantum-mechanical facts, and facts about virtual tables supervene on computational facts. So it seems reasonable to say that tables are constituted by quantum processes, and that virtual tables are constituted by computational processes. Further specificity in either case depends on delicate questions of metaphysics.

Reflecting on the third-person case, in which we are looking at a brain in a vat in our world, one might protest that virtual objects don't really exist: there aren't real *objects* corresponding to tables anywhere inside a computer. If one says this, though, one may be forced by parity into the view that tables do not truly exist in our quantum-mechanical world. If one adopts a restricted ontology of objects in one case, one should adopt it in the other; if one adopts a liberal ontology in one case, one should adopt it in the other. The only reasonable way to treat the cases differently is to adopt a sort of contextualism about what counts as an "object" (or about what falls within the domain of a quantifier such as "everything"), depending on the context of the speaker. But this will just reflect a parochial

fact about our language, rather than any deep fact about the world. In the deep respects, virtual objects are no less real than ordinary objects.

#### Note 11

The response to objection 8 is reminiscent of the familiar point, associated with Russell and Kant, that we do not know the intrinsic nature of entities in the external world. When it comes to physical entities, perception and science may tell us how these entities affect us, and how they relate to each other, but these methods tell us little about what the fundamental physical entities are like in themselves. That is, these methods reveal the causal structure of the external world, but they leave its intrinsic nature open.

The Metaphysical Hypothesis is in part a hypothesis about what underlies this microphysical causal structure: microphysical entities are made of bits. The same goes for the Matrix Hypothesis. One might say that if we are in a matrix, the Kantian ding-an-sich (thing in itself) is part of a computer-an-sich. This hypothesis supplements our ordinary conception of the external world, but it does not really contradict it, as this ordinary conception is silent on the world's intrinsic nature.

## Note 12

One general moral is that the "manifest image" is *robust*: our ordinary conception of the macroscopic world is not easily falsified by discoveries in science and metaphysics. As long as the physical world contains processes with the right sort of causal and counterfactual structure, then it will be compatible with the manifest image. Even a computer simulation has the relevant causal and counterfactual structure, as does a process in the mind of God. This is why they can support a robust external reality, despite their surprising nature.

This sort of flexibility in our conception of the world is closely tied to the semantic nonneutrality of many of our concepts. Those concepts, such as 'water', 'hair', and 'electron', leave some flexibility in what their referent might turn out to be. We conceive of their referents roughly as whatever actual entity plays a certain causal role, or has a certain appearance, while leaving open their intrinsic nature. One can likewise argue that the strongest constraints imposed by our conception of the world are plausibly those associated with semantically neutral concepts, which do not yield this sort of flexibility. These concepts plausibly include many of our causal (and nomic) concepts, as well as many of our mental concepts. In these cases, we have a sort of "direct" grasp of how the world must be in order to satisfy the concepts. If so, then our causal and mental beliefs impose strong constraints on the way the actual world must be. One can argue that our fundamental semantically neutral concepts are mental concepts ('experience', 'belief'), causal concepts ('cause', 'law'), logical and mathematical concepts ('and', 'two'), and categorical concepts ('object', 'property'). There are also many semantically neutral concepts that involve more than one of these elements: 'friend', 'action', and 'computer' are examples. If this is right, then the fundamental constraints that our beliefs impose on the external world are that it contains relevant mental states (in ourselves and in others) and that it contains objects and properties that stand in relevant causal relations to each other and to the mental states. This sort of conception is weak enough that it can be satisfied by a matrix (at least if it is a multivat matrix, or if computationalism about the mind is true).

In my opinion, this issue about the fundamental constraints that our beliefs impose on the world is the deepest philosophical issue that arises from thinking about the matrix. If what I have said in this article is right, it is precisely because these constraints are relatively weak that many hypotheses that one might have thought of as "skeptical" turn out to be compatible with our beliefs. And it is this that enables us to mount some sort of response to the skeptical challenge. A little paradoxically, one might say that it is because we demand so little that we know so much.

## Note 13

Why does a computer simulation of a world satisfy these constraints? The reason is tied to the nature of computation and implementation. Any formal computation can be regarded as giving a specification of (abstract) causal structure, specifying the precise manner of interaction between some set of formal states. To implement such a formal computation, it is required that the implementation have concrete states that map directly onto these formal states, where the pattern of (causal and counterfactual) interaction between these states precisely mirrors the pattern of interaction between the formal states (see Chalmers 1994). So any two implementations of the computation will share a certain specific causal structure. A computational description of the physical world will be required to mirror its causal structure down to the level of fundamental objects and properties. So any implementation of this computation will embody this causal structure (in transitions between implementing states, whether these be voltages, circuits, or something quite different). So insofar as our conception of the external world imposes constraints on causal structure that a real physical world can satisfy, these constraints will also be satisfied by a computer simulation. (This relates to a point made by Hubert and Stephen Dreyfus in their chapter in this collection. Like me, the Dreyfuses take the view that most of the beliefs of the inhabitants of a matrix will be true, not

false. But the Dreyfuses suggest that many of their causal beliefs will be false: for example, their general belief that "a physical universe with causal powers makes things happen in our world", and perhaps their specific beliefs that germs cause disease, that the sun causes things to get warm, and so on. On my view, this suggestion is incorrect. On my view, the world of someone living in a matrix has real causation going on everywhere within it, grounded in the real causation going on in the computer. Virtual germs in the computer really do cause virtual disease in the computer. So when matrix inhabitants say, "Germs cause disease", what they say is true.)

Of course the mental constraints also need to be satisfied. In particular, it is important that the causal structure stand in the right sort of relation to our experiences. But this constraint will also be satisfied when we are hooked up to a matrix. Constraints regarding other minds will be satisfied as long as we are in a multi-vat matrix, or if computationalism about the mind is true. In this way, a matrix has everything that is required to satisfy the crucial causal and mental constraints on our conception of the world.

## Note 14

A possible line of objection to the argument in this paper is to chapter somehow that there are *further constraints* that our beliefs impose on the world that the Matrix Hypothesis does not satisfy. One could argue that a mere match in mental and causal structure is not enough. For example, one might argue that the world needs to have the right *spatial* properties, where we have some sort of direct grip on what spatial properties are (perhaps because spatial concepts are semantically neutral). And one could suggest that the problem with the matrix is that its spatial properties are all wrong. We believe that external entities are arranged in a certain spatial pattern, but no such spatial pattern exists inside the computer.

In response, one can argue that these further constraints do not exist. It can be argued that spatial concepts are not semantically neutral, but instead are subject to Twin Earth thought-experiments. My student Brad Thompson has developed thought-experiments of this sort (Thompson 2003), involving a Doubled Earth where 'one meter' refers to (what we call) two meters, an El Greco World where 'square' refers to (what we call) rectangles, and so on. On this view, our spatial concepts pick out whatever manifold of properties and relations in the external world is causally responsible for our corresponding manifold of spatial experiences: in this respect, spatial concepts are analogous to color concepts. Here we do not have any "direct" grip on the basic nature of spatial properties. Instead, once again, the basic constraints are mental and causal.

This line of objection is tacitly engaged in section 6 of this chapter, where I suggest that if there is a computational level underneath physics, then any implementation of the relevant formal computation could serve in principle as a realization of that level, without compromising physical reality. Perhaps opponents might deny that there could be a computational level underneath physics, or at least might hold that there are constraints on what sort of implementation can serve. For example, they might hold that the implementing level itself must have an appropriate spatial arrangement.

I think that this line of response runs counter to the spirit of contemporary physics, however. Physicists have seriously entertained the idea that space as we understand it is not fundamental, but that there is an underlying level, not described in terms of ordinary spatial notions, from which space emerges. The Cellular Automaton Hypothesis is just one such proposal. Here, what is crucial is simply a pattern of causal interaction. If physicists discover that this pattern is realized in turn by an entirely different sort of level with very different properties, they will not conclude that ordinary physical space does not exist. Rather, they will conclude that space is itself constituted by something nonspatial. This sort of discovery might be surprising and revisionary, but again no more so than quantum mechanics. And, as with quantum mechanics, we would almost certainly not regard it as a skeptical hypothesis about the macroscopic external world. If this is right, then our conception of the macroscopic world does not impose essentially spatial constraints on the fundamental level of reality.

Similar issues arise with respect to time. In one respect, time poses fewer problems than space, as the computer simulation in a matrix unfolds in time, in the same temporal order as time in the simulated world. So one cannot object that the relevant temporal arrangements are not present in the matrix, in the way that one could object that the relevant spatial arrangements are not present. So even if temporal concepts were semantically neutral, the Matrix Hypothesis could still vindicate our temporal beliefs. Still, I think one can make a case that our concept of external time is not semantically neutral (it is notable that physicists have entertained hypotheses on which temporal notions play no role at the fundamental level). Rather, it picks out the external manifold of properties and relations that is responsible for our corresponding manifold of temporal experiences. If so, then any computer simulation with the right causal structure and the right relation to our experience will vindicate our temporal beliefs, regardless of its intrinsic temporal nature.

## Note 15

The reasoning in this chapter does not offer a knockdown refutation of skepticism, as several skeptical hypotheses are left open. But I think it significantly strengthens one of the

standard responses to skepticism. It is often held that although various skeptical hypotheses are compatible with our experiences, the hypothesis that there is a real physical world provides a simpler or better explanation of the regularities in our experiences than these skeptical hypotheses. If so, then we may be justified in believing in the real physical world, by an inference to the best explanation.

At this point, it is often objected that some skeptical hypotheses seem just as simple as the standard explanation, for example, the hypothesis that all of our experiences are caused by a computer simulation, or by God. If so, this response to skepticism fails. But if I am right, then these "equally simple" hypotheses are not skeptical hypotheses at all. If so, then inference to the best explanation may work after all: all of these simple hypotheses yield mostly true beliefs about an external world.

The residual issue concerns the various remaining skeptical hypotheses on the table, such as the Recent Matrix Hypothesis, the Local Matrix Hypothesis, and so on. It seems reasonable to hold that these are significantly less simple than the hypotheses above, however. All of them involve a non-uniform explanation of the regularities in our experiences. In the Recent Matrix Hypothesis, present regularities and past regularities have very different explanations. In the Local Matrix Hypothesis, beliefs about matters close to home and about those far from home have very different explanations. These hypotheses as a whole have a sort of dualmechanism structure that seems considerably more complex than the uniform-mechanism structures above. If this is right, one can argue that inference to the best explanation justifies us in ruling out these hypotheses, and in accepting the nonskeptical hypotheses above.

Even ifone thinks that some of these skeptical hypotheses offer reasonably good explanations of our experience, there is still a promising argument against global externalworld skepticism in the vicinity. If I am right, all of these skeptical hypotheses are at worst *partial* skeptical hypotheses: if they are correct, then a good many of our empirical beliefs will still be true, and there will still be an external world. To obtain a *global* skeptical hypothesis, we have to go all the way to the Chaos Hypothesis. But this is a hypothesis on which the regularities in our experience have no explanation at all. Even an extremely weak version of inference to the best explanation justifies us in ruling out this sort of hypothesis. If so, then this sort of reasoning may justify our belief in the existence of the external world.

# **Further reading**

The pieces by Descartes and Putnam in the appendix develop the classic evil-demon and brain-in-a-vat scenarios, respectively. Nick Bostrom's article "Are You Living in a Computer Simulation?" contains reasoning that should make us take quite seriously the possibility that we really are in some sort of matrix. Stephen Wolfram's *A New Kind of Science* speculates on the possibility that computation may underlie physics.

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